

# Exploratory Studies in Generalized Predictive Control for Active Aeroelastic Control of Tiltrotor Aircraft

*Raymond G. Kvaternik and Jer-Nan Juang  
Langley Research Center, Hampton, Virginia*

*Richard L. Bennett  
Bell Helicopter Textron, Inc., Fort Worth, Texas*

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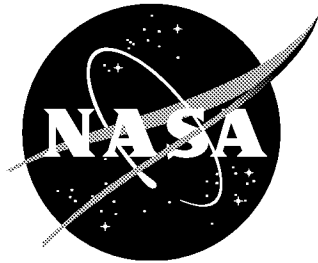
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*Raymond G. Kvaternik and Jer-Nan Juang*  
*Langley Research Center, Hampton, Virginia*

*Richard L. Bennett*  
*Bell Helicopter Textron, Inc., Fort Worth, Texas*

National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23681-2199

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Raymond G. Kvaternik and Jer-Nan Juang  
NASA Langley Research Center  
Hampton, VA

and

Richard L. Bennett  
Bell Helicopter Textron, Inc  
Fort Worth, TX

## Abstract

*The Aeroelasticity Branch (AB) at NASA Langley Research Center has a long and substantive history of tiltrotor aeroelastic research. That research has included a broad range of experimental investigations in the Langley Transonic Dynamics Tunnel (TDT) using a variety of scale models and the development of essential analyses. Since 1994, the tiltrotor research program has been using a 1/5-scale, semispan aeroelastic model of the V-22 designed and built by Bell Helicopter Textron Inc (BHTI) in 1983. That model has been refurbished to form a tiltrotor research testbed called the Wing and Rotor Aeroelastic Test System (WRATS) for use in the TDT. In collaboration with BHTI, studies under the current tiltrotor research program are focused on aeroelastic technology areas having the potential for enhancing the commercial and military viability of tiltrotor aircraft. Among the areas being addressed, considerable emphasis is being directed to the evaluation of modern adaptive multi-input multi-output (MIMO) control techniques for active stability augmentation and vibration control of tiltrotor aircraft. As part of this investigation, a predictive control technique known as Generalized Predictive Control (GPC) is being studied to assess its potential for actively controlling the swashplate of tiltrotor aircraft to enhance aeroelastic stability in both helicopter and airplane modes of flight. This paper summarizes the exploratory numerical and experimental studies that were conducted as part of that investigation.*

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*American Helicopter Society Northeast Region*  
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## Outline

- Introductory remarks
- Theoretical development
- Features characterizing method
- Computational considerations
- Implementation considerations
- Illustrative numerical and experimental results
- Concluding remarks
- Status and plans

## Some Relevant Background

- AB/TDT has a long history of tiltrotor aeroelastic research
- Research has included a broad range of experimental investigations and the development of essential analyses
- Since 1994, tiltrotor research program has been using a 1/5-scale, semispan aeroelastic model of the V-22
- AB/BHTI cooperative study of promising tiltrotor aeroelastic technology areas underway since 1994
- Evaluation of modern adaptive MIMO control techniques for active stability augmentation a major activity
- GPC-based method for actively controlling swashplate to enhance stability under investigation since 1997

The Aeroelasticity Branch (AB) at NASA Langley Research Center has a long and substantive history of tiltrotor aeroelastic research (ref. 1). That research has included a broad range of experimental investigations in the Langley Transonic Dynamics Tunnel (TDT) using a variety of scale models and the development of essential analyses. Since 1994, the tiltrotor research program has been using a 1/5-scale, semispan aeroelastic model of the V-22 designed and built by Bell Helicopter Textron Inc (BHTI) in 1983. That model has been refurbished to form a tiltrotor research testbed called the Wing and Rotor Aeroelastic Test System (WRATS) for use in the TDT. In collaboration with BHTI, studies under the current tiltrotor research program are focused on aeroelastic technology areas having the potential for enhancing the commercial and military viability of tiltrotor aircraft. Among the areas being addressed, considerable emphasis is being directed to the evaluation of modern adaptive multi-input multi-output (MIMO) control techniques for active stability augmentation and vibration control of tiltrotor aircraft. As part of this investigation, a predictive control technique known as Generalized Predictive Control (GPC) is being studied to assess its potential for actively controlling the swashplate of tiltrotor aircraft to enhance aeroelastic stability in both helicopter and airplane modes of flight. This paper summarizes the exploratory numerical and experimental studies that were conducted as part of that investigation.

## Approaches to Control

- Conventional control approach
  - System modeling and verification (off-line)
  - Controller design and verification (off-line)
- Adaptive control approach
  - On-line/real-time system identification
  - On-line/real-time controller design
  - User specification of model order, horizons, and control weights
- Autonomous control approach
  - System identification/controller design combined into one step
  - Automated estimation of system order and disturbances
  - Automated adjustment of control weights and control gains
  - All done on-line and in real-time

The approaches to active control may be grouped into three general categories: conventional control, adaptive control, and autonomous control.

Conventional approach: Uses an explicit plant model represented by a transfer function or state-space model. Determination and verification of the system's transfer function and the design of the controller are done off-line. LQR,  $H_\infty$ ,  $H_2$ , and  $\mu$ -synthesis are examples of such approaches.

Adaptive approach: The term adaptive as used here refers to a real-time digital system that updates the system identification (SID) parameters based on input/output data, uses the updated system parameters to compute a new set of control parameters, and then computes the next set of commands to be sent to the actuators, with all computations being done on-line. The user specifies the order of the system model, the prediction and control horizons, and the control weights.

Autonomous approach: SID and controller design are combined into one step. Estimation of system order and disturbances, and adjustment of control weights and control gains (e.g., via fuzzy logic, neural nets, and genetic algorithms) are all automated. All computations are done on-line and in real-time. Autonomous control is a very active area of research at this time.

The work addressed in this paper falls into the second category.



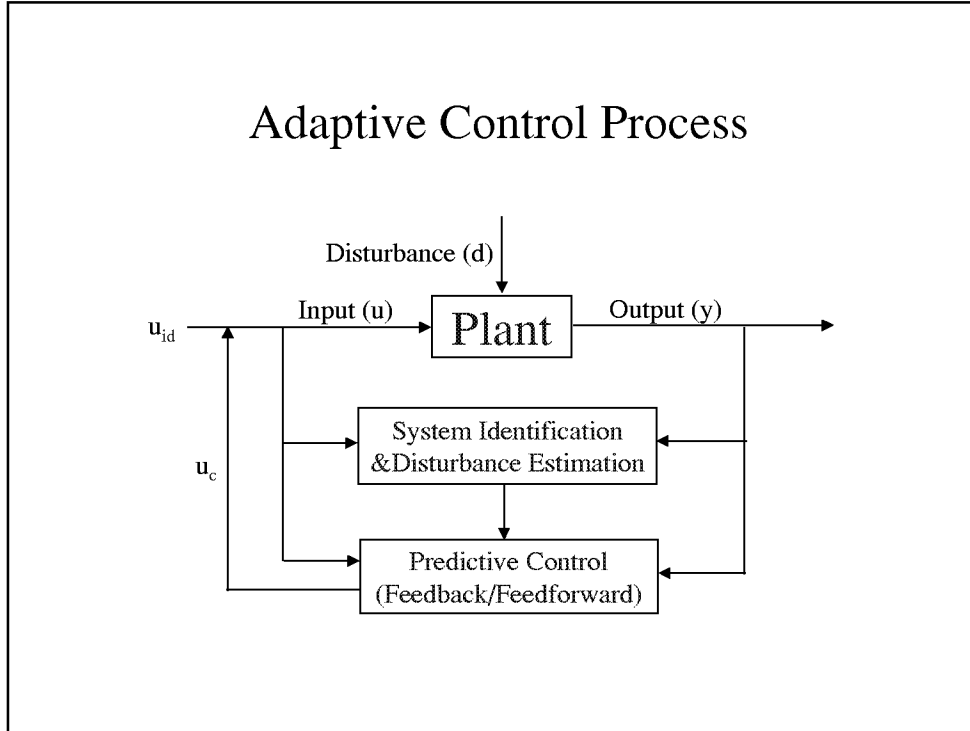
## Generalized Predictive Control

- Belongs to class of model-based predictive controllers used in process control industries since late 1970s
- Method introduced in 1987 by Clarke of Oxford University
- Input-output map of system used to form multi-step output prediction equation over a finite prediction horizon while subject to controls imposed over a finite control horizon
- Optimal control sequence determined by minimizing deviation of future system outputs from desired (target) response, subject to penalty on control power
- A novel version of GPC algorithm developed at NASA-Langley in 1997 for efficient computation and unknown disturbance rejection

Predictive controllers introduced in the chemical industries for controlling chemical processes have found applications in a wide variety of industrial processes (e.g., ref. 2). Predictive control refers to a strategy wherein the decision for the current control action is based on minimization of a quadratic objective function that involves a prediction of the system response some number of time steps into the future. A variety of predictive controllers have been proposed (e.g., ref. 3). Among these, Generalized Predictive Control (GPC), which was introduced in 1987 (refs. 4-5), has received notable attention by researchers. GPC is a time-domain multi-input-multi-output (MIMO) predictive control method that uses a finite-difference representation for the input-output map of the system. The input-output equation is used to form a multi-step output prediction equation over a finite prediction horizon while subject to controls imposed over a finite control horizon. The control to be imposed at the next time step is determined by minimizing the deviation of the predicted controlled plant outputs from the desired (or target) outputs, subject to a penalty on control effort.

A novel version of the GPC procedure was developed at NASA Langley Research Center in 1997 for efficient computation and unknown disturbance rejection by Dr. Jer-Nan Juang and his associates. Their work has resulted in a suite of MATLAB m-files that have been collected into a Predictive toolbox that can be used by researchers for GPC studies. A summary of the SID and control theory underlying their development is found in references 6-13, among others. These references were the primary sources of information for the work reported in this paper.

## Adaptive Control Process



The essential features of the adaptive control process used in the present GPC investigation are depicted in the diagram above. The system (plant) has  $r$  control inputs  $u$ ,  $m$  measured outputs  $y$ , and is subject to unknown external disturbances  $d$ . Measurement noise is also present. There are two fundamental steps involved: (1) identification of the system; and (2) use of the identified model to design a controller. A finite-difference model in the form of an Auto-Regressive moving average with eXogenous input (ARX) model is used here. This model is used for both system ID and controller design. System identification is done on-line in the presence of any disturbances acting on the system, as indicated in the center box of the diagram. In this way, an estimate of the disturbance model is reflected in the identified system model and does not have to be modeled separately. This approach represents a case of feedback with embedded feedforward. Because the disturbance information is embedded in the feedforward control parameters, there is no need for measurement of the disturbance signal (ref. 13). The parameters of the identified model are used to compute the predictive control law. A random excitation  $u_{id}$  (sometimes called dither) is applied initially with  $u_c$  equal to zero to identify the open-loop system. Dither is added to the closed-loop control input  $u_c$  if it is necessary to re-identify the system while operating in the closed-loop mode.

## Form of Model and Control Law Equations

- ARX model used for system identification

$$y(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \cdots + \alpha_p y(k-p) \\ + \beta_0 u(k) + \beta_1 u(k-1) + \cdots + \beta_p u(k-p)$$

- Control law equation

$$u_c(k) = \alpha_1^c y(k-1) + \alpha_2^c y(k-2) + \cdots + \alpha_p^c y(k-p) \\ + \beta_1^c u(k-1) + \beta_2^c u(k-2) + \cdots + \beta_p^c u(k-p)$$

The relationship between the input and output time histories of a MIMO system are described by the time-domain AutoRegressive eXogenous (ARX) finite-difference model shown in the first equation. This equation states that the current output  $y(k)$  at time step  $k$  may be estimated by using  $p$  sets of the previous output and input measurements,  $y(k-1), \dots, y(k-p)$  and  $u(k-1), \dots, u(k-p)$ ; and the current input measurement  $u(k)$ . The integer  $p$  is called the order of the ARX model. The coefficient matrices  $\alpha_i$  and  $\beta_i$  appearing in this equation are referred to as observer Markov parameters (OMP) or ARX parameters and are the quantities to be determined by the identification algorithm. Closed-loop robustness is enhanced by performing the system identification in the presence of the external disturbances acting on the system, thereby ensuring that disturbance information will be incorporated into the system model. The goal of SID is to determine the OMP based on input and output data. The OMP may be determined by any SID techniques that returns an ARX model of the system.

The ARX model is used to design the controller and leads to a control law that in the case of a regulator problem has the general form given by the second equation. This equation indicates that the current control input  $u_c(k)$  may be computed using  $p$  sets of the previous input and output measurements. The coefficient matrices  $\alpha_i^c$  and  $\beta_i^c$  appearing in this equation are the control gain matrices. The derivation of the parameters appearing in the SID and control law equations is described below.

## System Identification

- The digitized input and output time histories at  $l$  time points are used to form the data matrices  $y$  and  $V$  in the equation

$$y = \bar{Y} V$$

according to

$$y = \begin{bmatrix} y(0) & y(1) & y(2) & \cdots & y(p) & \cdots & y(l-1) \end{bmatrix}$$

$$V = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(p) & \cdots & u(l-1) \\ & v(0) & v(1) & \cdots & v(p-1) & \cdots & v(l-2) \\ & & v(0) & \cdots & v(p-2) & \cdots & v(l-3) \\ & & & \ddots & \vdots & \cdots & \vdots \\ & & & & v(0) & \cdots & v(l-p-1) \end{bmatrix}$$

$$[r + (r+m)p] \times l$$

System identification in the presence of the operational disturbances acting on the system is the first of the two major computational steps. The external disturbances acting on the system are assumed to be unknown (unmeasurable). The number of control inputs is  $r$  and the number of measured outputs is  $m$ . The order of the ARX model ( $p$ ) and the number of time steps ( $l$ ) must be specified. Some guidelines for their selection are given later.

The system is excited with band-limited white noise for SID. These random excitations are input to all  $r$  control inputs simultaneously and the  $m$  responses measured. The digitized input and output time histories ( $u$  and  $y$ ) at  $l$  time points are then used to form the data matrices  $y$  and  $V$  as indicated in the slide. The sizes of the resulting arrays are noted. The equation shown follows from writing the discrete-time state-space equations for a LTI system at a sequence of time steps  $k = 0, 1, \dots, l-1$  and grouping them into the above matrix form. The vector  $v$  is defined on the next slide.

In forming the matrices above, it has been assumed that the state matrix  $A$  is asymptotically stable so that for some sufficiently large  $p$ ,  $A^k \approx 0$  for all time steps  $k \geq p$ , and that an observer has been added to the system. It is through these expedients that the matrix  $V$  is reduced to a size amenable for practical numerical computation of its pseudo-inverse. The SID process yields OMP rather than system Markov parameters (SMP) because of the inclusion of an observer. A complete discussion of these aspects of the development may be found in reference 6.

## System Identification (Concluded)

- The vector  $v(k)$  in the matrix  $V$  is formed according to

$$v(k) = \begin{Bmatrix} u(k) \\ y(k) \end{Bmatrix}$$

$(r+m) \times 1$

- $\bar{Y}$  is the matrix of observer Markov parameters (OMP) which are to be identified and has the form

$$\bar{Y} = \begin{bmatrix} \beta_0 & \beta_1 & \alpha_1 & \beta_2 & \alpha_2 & \beta_3 & \alpha_3 & \cdots & \beta_p & \alpha_p \end{bmatrix}$$

$m \times r \quad m \times r \quad m \times m \quad m \times r \quad m \times m \quad m \times r \quad m \times m \quad m \times r \quad m \times m$   
 $m \times [r + (r+m)p]$

- The solution for  $\bar{Y}$  is given by

$$\bar{Y} = y V^\dagger = y V^T [V V^T]^{-1}$$

where  $\dagger$  denotes the pseudo-inverse.

The vector  $v(k)$  that appears in the data matrix  $V$  is formed from the vectors  $u(k)$  and  $y(k)$  as indicated above. Because of the introduction of an observer into the discrete state equations for the system (ref. 6), the quantities  $\alpha_i$  and  $\beta_i$  in  $\bar{Y}$  are referred to as observer Markov parameters (OMP). The solution for  $\bar{Y}$ , the vector of observer Markov parameters, is obtained by solving the equation  $y = \bar{Y} * V$  for  $\bar{Y}$  as shown in the bottom equation in the slide above. If the product  $VV^T$  is a well-conditioned matrix, the ordinary inverse can be taken as shown. Otherwise, a pseudo-inverse must be used. It should be noted that because the size of  $VV^T$  is much smaller than  $V$ , a pseudo-inverse may be appropriate even if the product is well conditioned.

## Derivation of Multi-Step Output Prediction Equation

The input-output map is given by the ARX equation

$$y(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \cdots + \alpha_p y(k-p) \\ + \beta_0 u(k) + \beta_1 u(k-1) + \cdots + \beta_p u(k-p)$$

Using this equation, the output at time step  $k+j$  can be written as

$$y(k+j) = \alpha_1^{(j)} y(k-1) + \alpha_2^{(j)} y(k-2) + \cdots + \alpha_p^{(j)} y(k-p) \\ + \beta_0 u(k+j) + \beta_0^{(1)} u(k+j-1) + \cdots + \beta_0^{(j)} u(k) \\ + \beta_1^{(j)} u(k-1) + \beta_2^{(j)} u(k-2) + \cdots + \beta_p^{(j)} u(k-p)$$

where the coefficient matrices are combinations of  $\alpha_i$  and  $\beta_i$

The one-step ahead ARX equation describing the input-output map of the system is the starting point for deriving the multi-step output prediction equation that is needed for designing the MIMO controller. Using the one-step ahead equation shown at the top of the slide, the output at time step  $k+j$  may be written in the form shown in the second equation, where the coefficient matrices are given by recursive expressions involving the matrices  $\alpha_i$  and  $\beta_i$  appearing in the ARX equation (ref. 10). The matrices  $\alpha_i$  and  $\beta_i$  are determined by the system identification process described earlier.

The second equation states that the output  $y(k+j)$  at time step  $k+j$  may be estimated by using  $p$  sets of the previous output and input measurements,  $y(k-1), \dots, y(k-p)$  and  $u(k-1), \dots, u(k-p)$ , and the (unknown) current and future inputs,  $u(k), u(k+1), \dots, u(k+j)$ . Letting  $j$  range over the set of values  $j = 1, 2, \dots, h_p-1$ , the resulting equations can be assembled into a multi-step ahead output prediction equation having the form indicated in the next slide.

## Derivation of Prediction Equation (Concluded)

Letting  $j$  range over the set of values  $j = 1, 2, \dots, q, \dots, h_p-1$ , the resulting equations can be assembled into the multi-step output prediction equation:

$$\begin{array}{ccccccc}
 \begin{array}{c} \text{Current \& Future Output} \\ \downarrow \end{array} & & \begin{array}{c} \text{Current \& Future Input} \\ \downarrow \end{array} & & \begin{array}{c} \text{Past Input} \\ \downarrow \end{array} & & \begin{array}{c} \text{Past Output} \\ \downarrow \end{array} \\
 y_{h_p}(k) = & \mathcal{T} & u_{h_c}(k) + & \mathcal{B} & u_p(k-p) + & \mathcal{A} & y_p(k-p) \\
 \begin{array}{c} h_p m \times 1 \end{array} & \begin{array}{c} h_p m \times h_c r \end{array} & \begin{array}{c} h_c r \times 1 \end{array} & \begin{array}{c} h_p m \times p r \end{array} & \begin{array}{c} p r \times 1 \end{array} & \begin{array}{c} h_p m \times p m \end{array} & \begin{array}{c} p m \times 1 \end{array}
 \end{array}$$

where the coefficient matrices  $\mathcal{T}$ ,  $\mathcal{B}$ ,  $\mathcal{A}$  are formed from combinations of the observer Markov parameters  $\alpha_i$  and  $\beta_i$  appearing in the ARX equation describing the input-output map of the system, and where  $h_p$  is the prediction horizon,  $h_c$  is the control horizon, and  $h_c \leq h_p$ .

GPC is based on system output predictions over a finite horizon  $h_p$  known as the prediction horizon. To predict future plant outputs, some assumption needs to be made about future control inputs. In determining the future control inputs for GPC, it is assumed that the control is applied over a finite horizon  $h_c$  known as the control horizon. Beyond the control horizon, the control input is assumed to be zero. In GPC, the control horizon is always equal to or less than the prediction horizon.

The general form of the multi-step ahead prediction equation indicating the sizes of the constituent matrices is shown in this slide. The coefficient matrices  $\mathcal{T}$ ,  $\mathcal{B}$ , and  $\mathcal{A}$  are formed from combinations of the observer Markov parameters  $\alpha_i$  and  $\beta_i$  appearing in the ARX equation describing the input-output map of the system. The objective is to predict the output for  $h_p$  time steps ahead, given the input for  $h_c$  steps ahead, where  $h_c \leq h_p$ . The quantity  $y_{h_p}(k)$  is the vector containing the predicted future outputs, whereas  $u_{h_c}(k)$  is the vector containing the future control inputs yet to be determined. The quantities  $u_p(k-p)$  and  $y_p(k-p)$  are vectors containing the previous  $p$  sets of control inputs and outputs, respectively.

## Expanded Form of Multi-Step Output Prediction Equation

$$\begin{aligned}
 & \boxed{\beta_0^{(q)} = \beta_1^{(q-1)} + \alpha_1^{(q-1)} \beta_0} \\
 & \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+q-1) \\ y(k+q) \\ \vdots \\ y(k+h_p-1) \end{bmatrix} = \begin{bmatrix} \beta_o^{(1)} & \beta_o & & & \\ \vdots & \vdots & \ddots & & \\ \beta_o^{(q-1)} & \beta_o^{(q-2)} & \dots & \beta_o & \\ \beta_o^{(q)} & \beta_o^{(q-1)} & \dots & \beta_o^{(1)} & \\ \vdots & \vdots & \ddots & \vdots & \\ \beta_o^{(h_p-1)} & \beta_o^{(h_p-2)} & \dots & \beta_o^{(h_p-h_c)} & \end{bmatrix} \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+h_c-1) \end{bmatrix} \quad h_c < h_p \\
 & + \begin{bmatrix} \alpha_1^{(1)} & \alpha_2^{(1)} & \dots & \alpha_{p-1}^{(1)} & \alpha_p^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_1^{(q-1)} & \alpha_2^{(q-1)} & \dots & \alpha_{p-1}^{(q-1)} & \alpha_p^{(q-1)} \\ \alpha_1^{(q)} & \alpha_2^{(q)} & \dots & \alpha_{p-1}^{(q)} & \alpha_p^{(q)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_1^{(h_p-1)} & \alpha_2^{(h_p-1)} & \dots & \alpha_{p-1}^{(h_p-1)} & \alpha_p^{(h_p-1)} \end{bmatrix} \begin{bmatrix} y(k-1) \\ y(k-2) \\ \vdots \\ y(k-p+1) \\ y(k-p) \end{bmatrix} + \begin{bmatrix} \beta_1 & \beta_2 & \dots & \beta_{p-1} & \beta_p \\ \beta_1^{(1)} & \beta_2^{(1)} & \dots & \beta_{p-1}^{(1)} & \beta_p^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta_1^{(q-1)} & \beta_2^{(q-1)} & \dots & \beta_{p-1}^{(q-1)} & \beta_p^{(q-1)} \\ \beta_1^{(q)} & \beta_2^{(q)} & \dots & \beta_{p-1}^{(q)} & \beta_p^{(q)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta_1^{(h_p-1)} & \beta_2^{(h_p-1)} & \dots & \beta_{p-1}^{(h_p-1)} & \beta_p^{(h_p-1)} \end{bmatrix} \begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(k-p+1) \\ u(k-p) \end{bmatrix} \\
 & \boxed{\alpha_{p-1}^{(q)} = \alpha_p^{(q-1)} + \alpha_1^{(q-1)} \alpha_{p-1}} \quad \boxed{\beta_{p-1}^{(q)} = \beta_p^{(q-1)} + \alpha_1^{(q-1)} \beta_{p-1}}
 \end{aligned}$$

The expanded form of the multi-step output prediction equation for the case in which the control horizon is less than the prediction horizon is shown in this slide. The OMP  $\alpha_i$  and  $\beta_i$  determined in system identification form the first block rows of the coefficient matrices  $\mathcal{T}$ ,  $\mathcal{A}$ , and  $\mathcal{B}$  in the equation. The terms in the remaining rows are computed using the recursive relations indicated in the boxes (ref. 10). All terms in the above equation are known, except for the  $h_c$  sets of future commands, and the  $h_p$  sets of predicted responses. The goal of the GPC control algorithm is to determine the set of future commands  $u(k)$ ,  $u(k+1)$ , ...,  $u(k+h_c-1)$  that are required to achieve a desired predicted response  $y(k)$ ,  $y(k+1)$ , ...,  $y(k+h_p-1)$ .

It should be remarked that the system Markov parameters (SMP), which are commonly used as the basis for identifying discrete-time models for linear dynamical systems, form the first block column in the matrix  $\mathcal{T}$ ; the remaining block columns are formed from subsets of the SMP. The Markov parameters are the pulse response of a system and are unique for a given system. The discrete-time state-space matrices  $A$ ,  $B$ ,  $C$ , and  $D$  are embedded in the SMP.

The multi-step output prediction equation is used to define an objective function whose minimization with respect to  $u_{hc}(k)$  leads to the control law from which a vector of future control inputs can be computed using the  $p$  sets of previous control inputs and measured outputs.



## Derivation of Control Law

Define an error function  $\varepsilon$ :

$$\varepsilon = y_T(k) - y_{h_p}(k) = y_T(k) - \mathcal{T}u_{h_c}(k) - \mathcal{B}u_p(k-p) - \mathcal{A}y_p(k-p)$$

Define an objective function  $J$ :

$$J = \varepsilon^T R \varepsilon + u_{h_c}^T Q u_{h_c}$$

Minimize  $J$  with respect to  $u_{h_c}(k)$  :

$$u_{h_c}(k) = -(\mathcal{T}^T R \mathcal{T} + Q)^{\dagger} \mathcal{T}^T R (-y_T(k) + \mathcal{B}u_p(k-p) + \mathcal{A}y_p(k-p))$$

$h_c r \times 1$

Retain the first component (first  $r$  rows) of  $u_{h_c}(k)$  :

$$u_c(k) = -\gamma^c y_T(k) + \beta^c u_p(k-p) + \alpha^c y_p(k-p)$$

$r \times 1$

The predictive control law is obtained by minimizing the deviation of the predicted controlled response (as computed from the multi-step output prediction equation) from a specified target response over a prediction horizon  $h_p$ . To this end, one first defines an error function that is the difference between the desired (target) response  $y_T(k)$  and the predicted response  $y_{hp}(k)$  and forms an objective function  $J$  quadratic in the error and the unknown future controls. Two weighting matrices are included in the objective function:  $Q$  (symmetric and positive-definite) is used to limit the control effort and stabilize the closed-loop system;  $R$  (symmetric and positive-semidefinite) is used to weight the relative importance of the differences between the target and predicted responses. Typically,  $Q$  and  $R$  are assumed to be diagonal and for  $Q$  to have the same value  $w_c$  along its diagonal and  $R$  to have the same value  $w_r$  along its diagonal. Minimizing  $J$  with respect to  $u_{hc}(k)$  and solving for  $u_{hc}(k)$  gives the control sequence to be applied to the system over the next  $h_c$  time steps. The first  $r$  values (corresponding to the first future time step) are applied to the  $r$  control inputs, the remainder are discarded, and a new control sequence is calculated at the next time step.

The target response is zero for a regulator problem and non-zero for a tracking problem.  $Q$  must be tuned to ensure a stable closed-loop system. Typically,  $h_c$  is chosen equal to  $h_p$ . However,  $h_c$  may be chosen less than  $h_p$  resulting in a more stable and sluggish regulator.

## Features Characterizing Method

- Predictive controllers employ a strategy wherein current control actions are based on prediction of system response at a number of time steps into the future
- Approach allows one to work directly with an input-output (ARX) model of the system
- SID process employs an observer to enable numerical computation of pseudo-inverse for calculation of OMP
- Controller is thus inherently observer-based but no explicit consideration of its presence needs to be taken into account
- System identification is performed in the presence of any disturbances acting on system

Predictive controllers such as GPC employ a strategy wherein the values of the current control inputs are based on the predicted responses at a number of time steps into the future. Such an approach lends itself to the use of an ARX model to represent the system for both system identification and controller design. The SID process used makes recourse to an observer to enable numerical computation of the pseudo-inverse needed for calculation of the OMP that comprise the coefficients of the ARX equation. The controller is thus inherently observer-based but no explicit consideration of the observer needs to be taken into account in the implementation.

In practice, the disturbances acting on the system are unknown or unmeasurable. However, as discussed in reference 13, by performing the SID in the presence of the external disturbances acting on a system, a disturbance model is implicitly incorporated into the identified observer Markov parameters. However, the identified model must be larger than the true system model to accommodate the unknown disturbances.

## Features Characterizing Method (Continued)

- Disturbance model is implicitly incorporated into the identified coefficients (OMP) of the ARX model
- Effects of unknown disturbances embedded in coefficient matrices  $\mathcal{A}$  and  $\mathcal{B}$  (and hence  $\alpha^c$  and  $\beta^c$ ) because SID done with disturbances acting on system
- Estimated order of ARX model used for system ID given by

$$p \geq \text{ceil} \left( \frac{n_{\text{system states}} + n_{\text{disturbance states}}}{m} \right)$$

- Prediction and control horizons are set according to

$$h_p \geq p \qquad h_c \leq h_p$$

The disturbance model is implicitly incorporated into the identified coefficients (OMP) of the ARX model. Thus, the effects of the unknown disturbances acting on the system are embedded in the matrices  $\mathcal{A}$  and  $\mathcal{B}$ , and hence the control law matrices  $\alpha^c$  and  $\beta^c$ .

An expression for estimating the order of the ARX model that is to be used for SID is given in the slide. The number of system states is typically chosen to be twice the number of significant structural modes; the number of disturbance states is set to twice the number of frequencies in the disturbance;  $m$  is the number of output measurements. If measurement noise is of concern, the order of  $p$  so computed should be increased to allow for computational poles and zeros to improve system identification in the presence of noise. In practice, simply choosing  $p$  to be 5-6 times the number of significant modes in the system is often adequate.

The prediction and control horizons are set according to the relations indicated at the bottom of the slide. Although  $h_p$  can be set equal to  $p$ ,  $h_p$  is typically set greater than  $p$  to help limit control effort so as not to saturate the control actuators. If the control horizon is greater than the system order a minimum energy (minimum norm) solution is obtained wherein the commanded output is shared so that the control actuators don't fight each other. If  $h_p$  is set equal to  $p$  one obtains a so-called deadbeat controller (ref. 10).

## Features Characterizing Method (Concluded)

- Extending  $h_c$  and  $h_p$  (and hence  $p$ ) to very large values, GPC solution approaches LQR solution; hence GPC approximates an optimal controller for large  $p$
- Matrices  $Q$  and  $R$  are control effort and tracking error weighting matrices
- Matrix  $\mathcal{T}$  (and hence  $\gamma^c$ ) is invariant if system does not change because it is formed from SMP (which are unique)
- Solution for  $\bar{Y}$  obtained by an efficient computational procedure

By extending the prediction and control horizons (and hence  $p$ , the order of the ARX model) to very large values, the GPC solution approaches that of the linear quadratic regulator (LQR). Thus, GPC approximates an optimal controller for large  $p$  (ref. 5).

Weighting matrices  $Q$  and  $R$  are used to limit the control effort and to weight the relative importance of the differences between the target and predicted responses, respectively. As mentioned earlier,  $Q$  and  $R$  are usually assumed to be diagonal and for  $Q$  to have the same value  $w_c$  along its diagonal and  $R$  to have the same value  $w_r$  along its diagonal. The GPC solution offers no guarantee of stability and the control weight must be tuned to produce an acceptable solution without going unstable. Reducing  $w_c$  increases control authority and performance but eventually drives the system unstable.

If only the disturbances acting on the system change, there is no need to recalculate  $\mathcal{T}$  (and hence  $\gamma^c$ ) because it is formed solely from the SMP, which are unique for a given system (ref. 13).

The solution for  $\bar{Y}$  described earlier involves forming the matrix products  $yV^T$  and  $VV^T$ . Here, these products are obtained using the computationally efficient procedure described in reference 8.

## GPC Computational Considerations

- System ID in presence of external disturbances for implicit inclusion of disturbance model
- Pseudo-inverse based on SVD techniques
- Basis for recalculation of matrices
- On-line/real-time versus on-line/near-real-time
- Batch versus recursive calculation of OMP
- Efficient computational algorithms and coding

Several considerations dealing with computations should be kept in mind when developing algorithms for GPC applications.

SID should be done with the external disturbances acting on the system so that information about the disturbances is embedded in the OMP. However, depending on the nature of the external disturbances, it may be possible to perform a SID on an undisturbed system and still determine a control law that results in satisfactory closed-loop performance.

The computation of pseudo-inverses should be performed using Singular Value Decomposition (SVD) because of the latter's ability to deal with matrices that are numerically ill-conditioned. The use of pseudo-inverses (via SVD) is recommended even in cases where the ordinary inverse may seem appropriate (such as in the operation  $(VV^T)^{-1}$  indicated in the expression for the OMP).

The basis for recalculation of matrices needs to be selected. For example, should the system be re-identified and the control law matrices recalculated in every sampling period, every specified number of time steps (block updating), or only if some event requiring a re-ID occurs? Should all calculations be done in real time, or can some be done in near real time? Should updating of the OMP be done in batch mode or recursively?

Microprocessor speeds are such that it is now often possible to complete the full cycle of GPC computations and apply the commands to the actuators within one sampling period. However, the need for efficient computational algorithms and attendant coding is not expected to diminish.

## GPC Implementation Considerations

- Low-pass filtering of measured time histories
- Cut-off frequency of filter set to Nyquist frequency  $f_N$
- Nyquist frequency chosen so that maximum frequency of interest is about 75% of  $f_N$
- Sampling frequency selected to be between 2-3 times  $f_N$
- Length of data stream for SID determined by need for 5-10 cycles of lowest frequency mode in measured responses
- Scaling (normalization) of digitized input and output data
- Distribution of computing tasks among computers/CPU's
- Extent of user interaction/involvement
- Constraints on acceptable values of input and output

Several considerations must be taken into account when actually implementing GPC algorithms in hardware for active controls work.

The measured response time histories must be passed through a low-pass filter with a cut-off frequency  $f_c$  set equal to the Nyquist frequency  $f_N$ . The latter is chosen so that the maximum frequency of interest is about 75% of  $f_N$ . The sampling frequency  $f_s$  should be at least twice  $f_N$  to prevent aliasing. However, if  $f_s$  is made too large the low frequency modes will be poorly identified due to a loss of frequency resolution. A sampling rate between 2 to 3 times  $f_N$  is generally sufficient. Once the sampling frequency has been selected, the minimum number of data points that should be used for SID follows from the requirement of having 5-10 cycles of the lowest frequency mode in the measured response time histories.

Normalization of the input and output data that is used for SID on the maximum actual or expected values of the data is often helpful numerically. The procedure will depend on whether the computations are being done in batch mode or recursively.

The computing tasks can be distributed among computers or different CPU's on a single computer. The choice will influence the extent of user involvement.

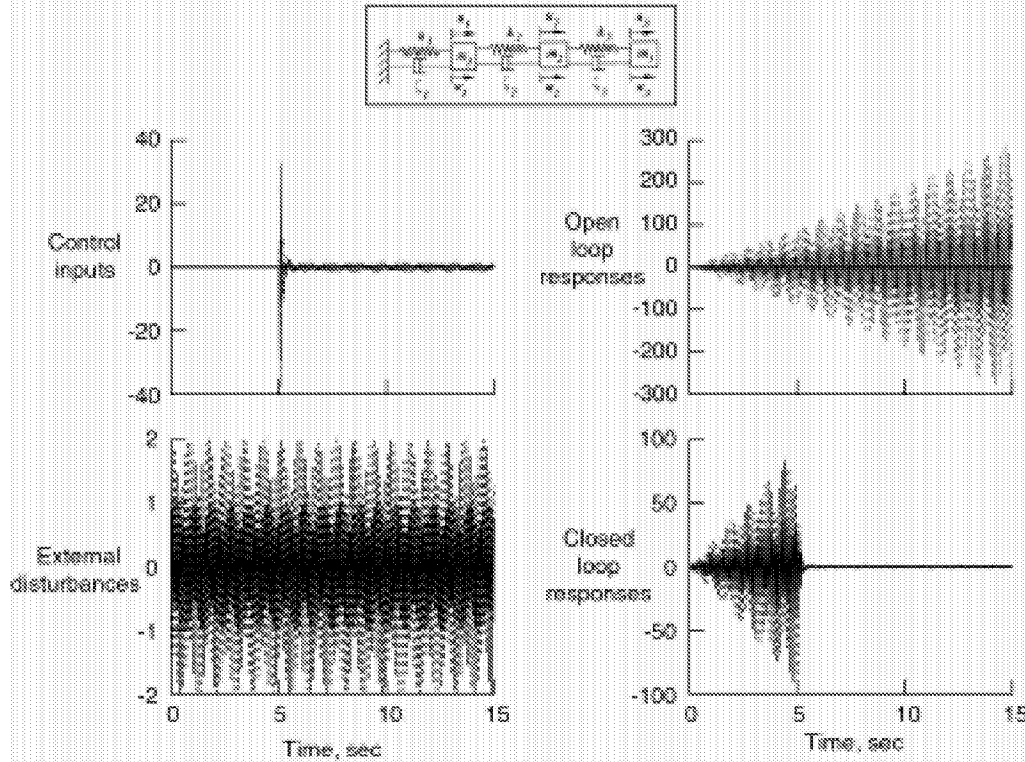
The values of the input and output data that are being used during closed-loop operations must be carefully monitored to ensure that they fall within acceptable bounds. This is easily done using IF/THEN-type checks in the code.

## Illustrative Numerical and Experimental Results

- Simulations using lumped-mass-spring-dashpot systems
- Experimental dynamic studies using Cobra stick model
- Simulated tracking control of XV-15 in roll maneuver
- Hover test of stiff-inplane rotor on WRATS
- Ground resonance test of soft-inplane rotor on WRATS

A variety of numerical and experimental studies comprised the exploratory evaluation phase of the GPC investigation during the period 1997-1999. These studies included: (1) Numerical simulations using 3- and 9-DOF mass-spring-dashpot systems varying the number and location of external disturbances, the number and location of control inputs, and the number, location, and type of response measurements (displacement, acceleration); (2) Extensive experimental simulations using the BHTI Cobra stick model in which the number and location of the shakers used for control and disturbance were varied; (3) Preliminary assessment of GPC for simulated tracking response using an XV-15 math model determined using BHTI's COPTER (COMprehensive Program for Theoretical Evaluation of Rotorcraft) flight simulation program (ref. 14); (4) A hover test (April 1998) of a stiff-inplane rotor on the WRATS testbed to explore the vibration reduction capability of GPC (excitation caused by rotor downwash and rpm near resonance); and (5) Ground resonance test (Oct. 1999) of a soft-inplane rotor on the WRATS testbed.

## SIMULATIONS FOR UNSTABLE 3-DOF SYSTEM



The 3-DOF mass-spring-dashpot system used in the 3-DOF simulations is depicted at the top of the slide.

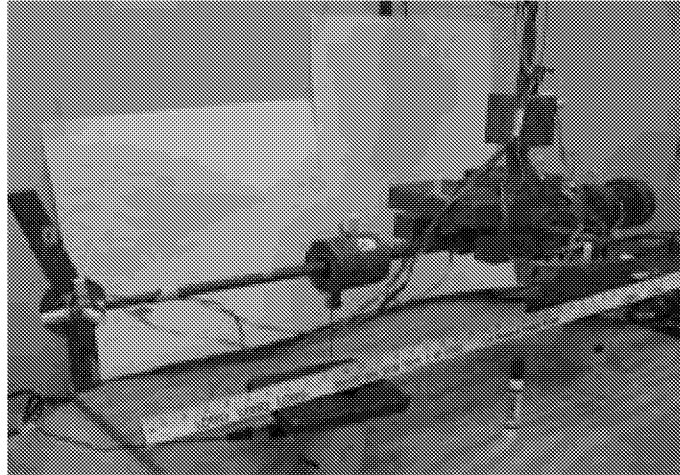
The masses were equal to 1.0 and the spring rates were all equal to 1000 lb/in. The damping coefficients of dashpots 1 and 2 were equal to 0.1 lb/in/sec, but the damping of dashpot 3 was set to  $-0.1$  lb/in/sec to make the system unstable (one pair of complex conjugate eigenvalues had a positive real part).

Control was imposed at masses 1 and 3 ( $r = 2$ ). Accelerations were measured at masses 1 and 2 ( $m = 2$ ). Sinusoidal disturbances were applied at masses 1 and 2 at a frequency  $f = 6.276$  Hz, which coincided with a natural frequency of the system. The expressions used to represent the disturbances in the simulation are  $ud1 = 2*\cos(2*\pi*f*t)$  and  $ud2 = 1*\sin(2*\pi*f*t)$  and are shown in the plot at the lower left. Also,  $\Delta t = 0.05$  sec,  $l = 300$ , and  $p = 5$ .

The oscillatory diverging behavior of the open-loop system is shown in the plot at the upper right, which shows the time history of the acceleration responses of masses 1 and 2. The loop was closed after 100 time steps. The time histories of the closed-loop responses and control commands are shown in the plots in the lower right and upper left, respectively.

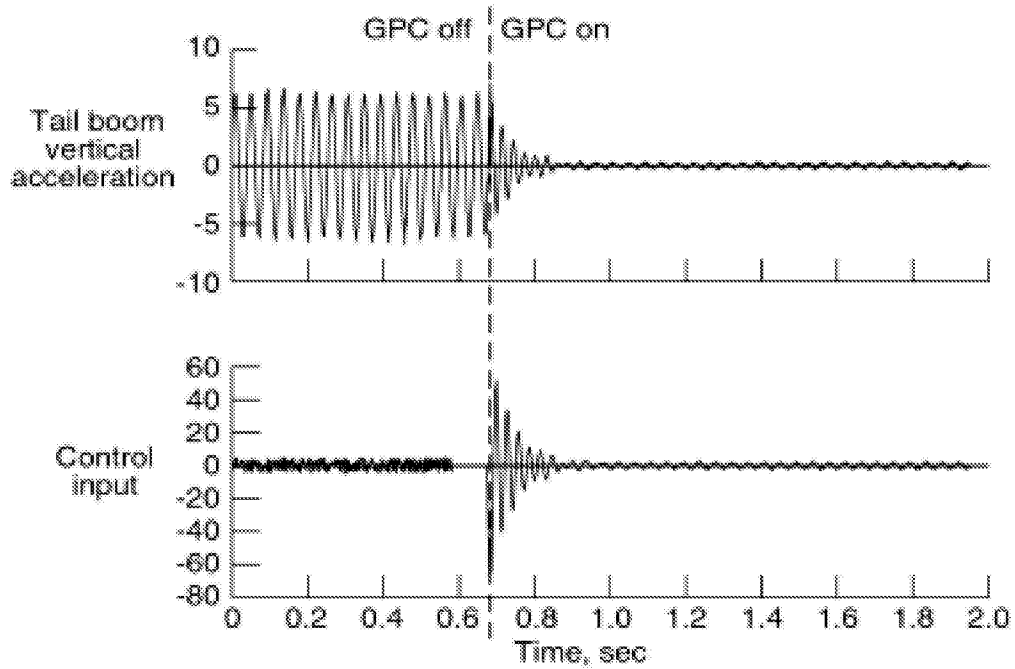


## Cobra Stick Model



In parallel with the extensive numerical simulations that were being conducted using the 3- and 9-DOF mass-spring-dashpot systems, bench tests were conducted by Bell Helicopter using the “Cobra stick model” shown in the slide. This model is a 36-inch long, 50-lb, multiple degree-of-freedom lumped-mass dynamic model that approximates the dynamics of a Cobra helicopter. The dominant vertical bending modes of the model have natural frequencies of about 8 and 23 Hz. Electromagnetic shakers were used in various combinations for imposing external periodic and random disturbances, random excitations needed for system identification, and the control inputs called for by the GPC algorithm.

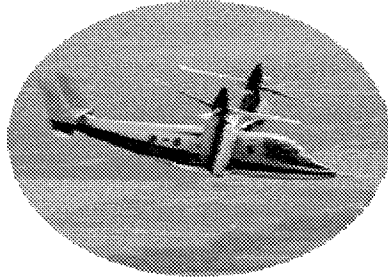
## EXPERIMENTAL RESULTS FROM COBRA STICK MODEL



An illustrative experimental result obtained using the Cobra stick model is shown in the slide above. The case shown is for sinusoidal excitation of 23.5 Hz (near a model natural frequency) at the nose of the model; random excitation at the main rotor hub for SID; vertical response at the tail rotor location; control input at the hub;  $f_s = 512$  Hz;  $l = 300$ ;  $m = 1$ ;  $r = 1$ ;  $p = 8$ ;  $w_c = w_r = 1.0$ . For the case shown here, 300 time points were used to ID the system and imposition of closed-loop control was then (arbitrarily) delayed for 50 time steps.

The top plot shows the time history of the tail boom vertical acceleration. The bottom plot shows the time history of the control input during SID, the delay, and after closing the loop. The response is reduced dramatically in about 0.2 sec.

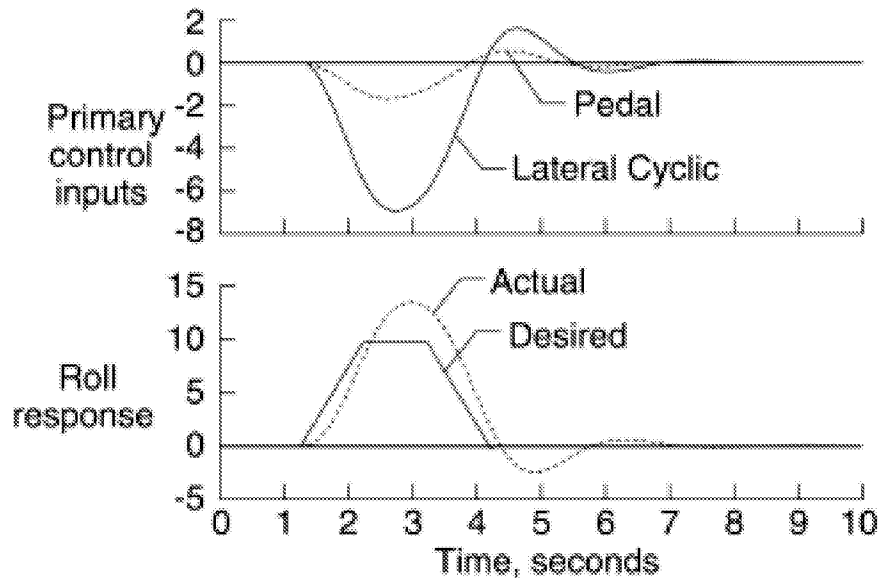
## Simulated Tracking Control of XV-15 Tiltrotor



- Aircraft in airplane mode of flight; 150 kts airspeed; sea level
- Control inputs: Collective, pedal, longitudinal & lateral cyclic pitch,
- Responses: Pitch, roll, sideslip, and forward velocity
- COPTER used to generate linearized mass, damping, and stiffness matrices for trimmed flight condition and components for  $B$  matrix
- MATLAB used to form  $A$ ,  $B$ ,  $C$ , and  $D$  state matrices and generate simulated input-output time history needed for SID/GPC
- Roll maneuver prescribed

A cursory examination of using the GPC algorithm as a means for actuating the controls necessary to execute a prescribed maneuver was made. The subject aircraft was the XV-15 cruising at 150 kts in the airplane mode of flight. The controls considered were collective pitch, longitudinal and lateral cyclic pitch, and pedal. Responses of interest were aircraft pitch, roll, sideslip, and forward velocity. Bell Helicopter's COPTER flight simulation program (ref. 14) was used to compute the linearized mass, damping, and stiffness matrices for a trimmed flight condition as well as components for the  $B$  matrix. MATLAB was used to form the  $A$ ,  $B$ ,  $C$ , and  $D$  state matrices and generate the simulated input-output time histories needed for SID and GPC. A one-cycle saw-tooth-type roll maneuver was prescribed to be the target response,  $y_T$ .

## SIMULATED ROLL TRACKING CONTROL OF XV-15 (Airplane Mode; 150 kts)

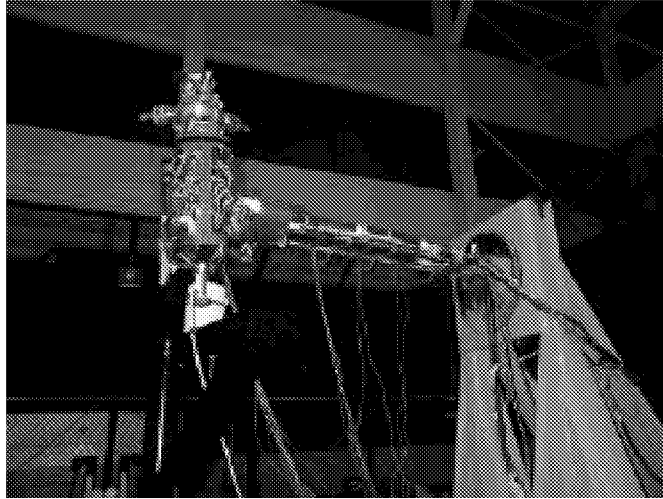


One cycle of a saw-tooth-type roll maneuver was prescribed to be the target response,  $y_T$ . The time histories of this desired response and the actual roll response produced by GPC are shown for comparison at the bottom of the slide. The remaining three responses (pitch, sideslip, and forward velocity) had negligible magnitudes and are not plotted.

The time histories of the rudder and lateral cyclic control inputs called for by the GPC algorithm to perform the prescribed roll maneuver are shown at the top of the slide. The remaining control inputs (collective pitch and longitudinal cyclic pitch) had negligible magnitudes and are not plotted.

The primary structural natural frequencies of the aircraft lie between 0.8 and 10 Hz. The values of pertinent GPC parameters are:  $w_c = 1.0$ ,  $p = 8$ ,  $h_p = h_c = 2p$ ,  $l = 400$ ,  $\Delta t = .025$  sec,  $f_s = 40$  Hz.

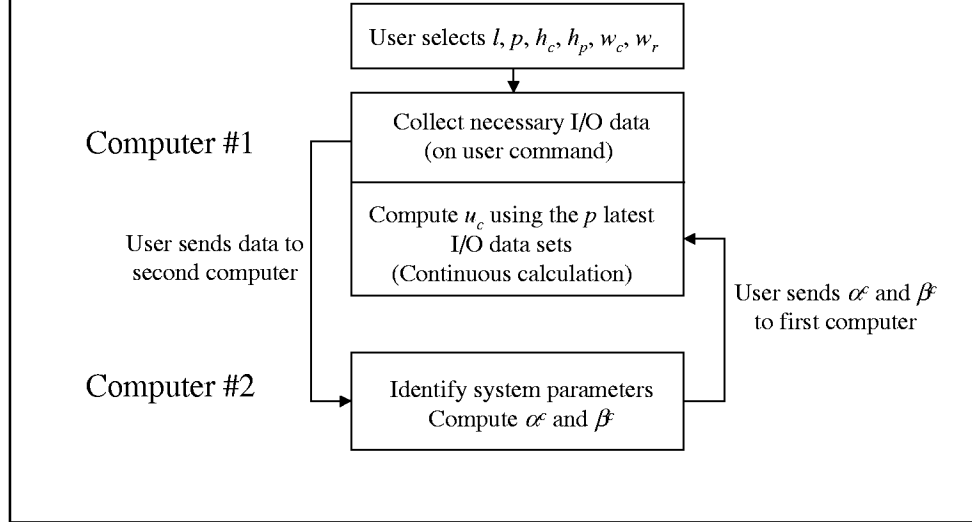
## WRATS Model in Aeroelasticity Branch Rotorcraft Hover Test Facility



The initial evaluation of GPC on the WRATS testbed was conducted in April 1998 during a one-week hover dynamics test conducted in the AB rotorcraft hover test facility located in a building adjacent to the TDT. A stiff-inplane gimbaled rotor was employed in this investigation. The model is shown above with its rotor blades removed and replaced by equivalent lumped weights for a ground vibration test that was conducted prior to the test. Emphasis in this initial evaluation of GPC was on active control of vibration using only the collective control. To provide a rigorous test of the GPC algorithm, the open-loop response of the model was exaggerated by running the rotor at an rpm that nearly coincided with the natural frequency of the wing vertical bending mode. Additional excitation of the wing was provided by the downwash associated with running the rotor at a high thrust level.

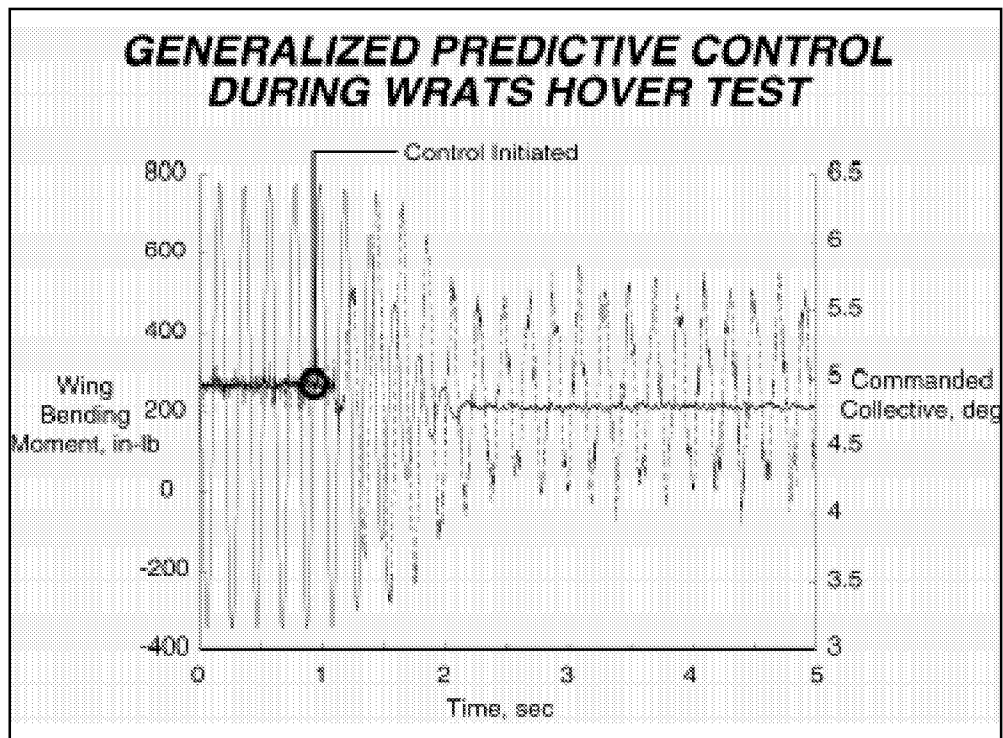
# Implementation of GPC Algorithm for WRATS Hover Dynamics Test

Dual-computer approach with user interaction:



For the hover dynamics investigation, the GPC computations were distributed between two computers and user interaction was required to transfer the data from one computer to the other. On user command, data required for system identification was collected on computer #1 and sent to computer #2 where SID was performed and the control gain matrices  $\alpha^c$  and  $\beta^c$  were computed. On user command the control parameters were sent to computer #1 which used the  $p$  latest data sets to (continuously) compute the control commands that were sent to the swashplate actuators. If re-identification of the system was required, the process was repeated on user command.

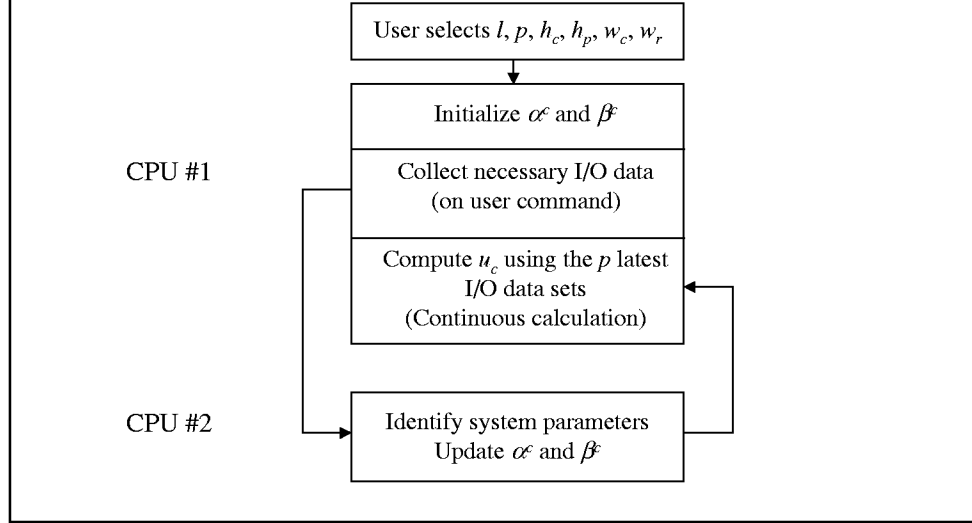
All computations were done using MATLAB on PCs with 500 MHz CPUs.



Some experimental results obtained during the WRATS hover dynamics test that illustrate the effectiveness of GPC in reducing rotor-induced vibrations are shown in this slide, which shows the measured time histories of the open- and closed-loop responses of the vertical bending moment rear the root of the wing. It is seen that the response is dramatically reduced within a second after the control is turned on.

## Implementation of GPC Algorithm for WRATS Ground Resonance Test

Single-computer/dual-processor approach:



For the ground resonance investigation, the GPC computations were distributed between two CPUs on a single computer with no user interaction required for data transfer as in the earlier hover test. On user command, data required for system identification was collected on CPU #1 and sent to CPU #2 where SID was performed and the control gain matrices  $\alpha^c$  and  $\beta^c$  were computed. The control parameters were automatically sent to CPU #1 which used the  $p$  latest data sets to (continuously) compute the control commands to be sent to the swashplate actuators. If re-identification was required, the process was repeated on user command.

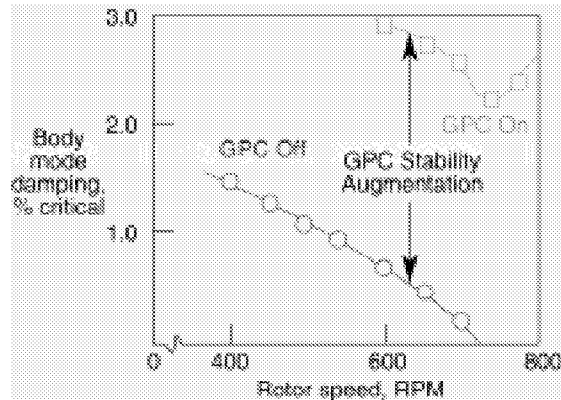
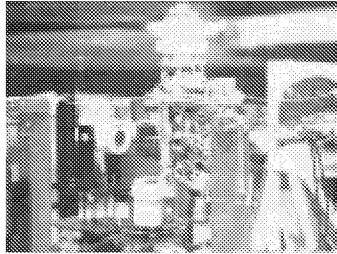
All the algorithms were implemented in dSPACE software on a PC with 500 MHz CPUs.



## Ground Resonance Test of Soft-Inplane Rotor on WRATS Testbed

(8 degrees collective at 75% r/R)

WRATS model in hover test cage.



A brief investigation of the use of GPC to actively control the ground resonance behavior of a soft inplane tiltrotor was conducted in the AB hover test facility in October 1999. For this test, the model blades were modified by replacing the stiff inplane flexure at the root of each blade with a spindle incorporating a lag hinge and an adjustable viscous damper and spring. The open-loop behavior (frequency and damping versus rotor speed) was compared with its closed-loop behavior for several values of collective pitch. A GPC-based algorithm was used to actively control the cyclic inputs into the (fixed-system) swashplate in a manner which produced a whirl of the rotor tip-path-plane in the direction and frequency needed to stabilize the critical body mode. For the open-loop configurations of the model in which a definitive ground resonance instability was observed, use of GPC was found to be strongly stabilizing. In particular, damping levels of about 2% critical were noted in the rpm range where the open-loop system was unstable. The results for the configuration with eight-degrees of collective pitch are shown at the right in the slide above. A photo of the model obtained from the video recording made during the test is at the left. A pneumatically-actuated snubber system, consisting of four horizontal cables attached to the upper end of the pylon in an X-pattern, was used to arrest instability and is partially visible in the photo.

## Concluding Remarks

- Exploratory studies of GPC for active aeroelastic control of tiltrotor aircraft completed
- Studies included numerical simulations and experimental investigations
- GPC-based MIMO active control system was highly effective in increasing stability and reducing response
- GPC appears to be a viable approach for active stability augmentation and vibration control of tiltrotor aircraft
- Wind-tunnel tests of WRATS model with GPC needed to more fully evaluate method

Exploratory numerical and experimental studies into the use of GPC for active aeroelastic control of tiltrotor aircraft have been completed. A GPC-based MIMO active control system was demonstrated to be highly effective in increasing ground resonance stability and reducing vibratory response. While these results are quite encouraging with respect to establishing the viability of the method, it is recognized that a broader evaluation of the methodology is needed to validate GPC-based algorithms for active stability augmentation and vibration control of tiltrotor aircraft.

## Status and Plans

- Initial wind-tunnel evaluation of GPC on WRATS for airplane-mode stability augmentation completed
- Method highly effective in increasing stability (damping) of critical wing mode
- Continue development and evaluation of GPC-based method for active aeroelastic control
- Improve suite of computational algorithms
- Establish robustness to system nonlinearities
- Conduct additional ground and wind-tunnel tests

The initial wind-tunnel evaluation of GPC on the WRATS testbed for airplane mode stability augmentation has recently (April 2000) been completed. The method was highly effective in increasing stability (damping) of the critical wing mode for all of the model conditions tested.

Plans are to continue development and evaluation of GPC for active aeroelastic control. Emphasis will be on improving the suite of computational algorithms comprising the current GPC software system developed for WRATS. In this regard, work is underway on providing for the calculation of the closed-loop eigenvalues using closed-loop input-output data and for the recursive calculation of the OMP.

It is expected that work will also be initiated to establish the robustness of GPC-based methods to system nonlinearities. Both numerical simulations and experimental studies are anticipated.

Additional ground and wind tunnel tests are to be conducted as necessary to evaluate the GPC-based methodology over a broader range of simulated flight and operating conditions.

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13. ABSTRACT (Maximum 200 words) The Aeroelasticity Branch at NASA Langley Research Center has a long and substantive history of tiltrotor aeroelastic research. That research has included a broad range of experimental investigations in the Langley Transonic Dynamics Tunnel (TDT) using a variety of scale models and the development of essential analyses. Since 1994, the tiltrotor research program has been using a 1/5-scale, semispan aeroelastic model of the V-22 designed and built by Bell Helicopter Textron Inc. (BHTI) in 1981. That model has been refurbished to form a tiltrotor research testbed called the Wing and Rotor Aeroelastic Test System (WRATS) for use in the TDT. In collaboration with BHTI, studies under the current tiltrotor research program are focused on aeroelastic technology areas having the potential for enhancing the commercial and military viability of tiltrotor aircraft. Among the areas being addressed, considerable emphasis is being directed to the evaluation of modern adaptive multi-input multi-output (MIMO) control techniques for active stability augmentation and vibration control of tiltrotor aircraft. As part of this investigation, a predictive control technique known as Generalized Predictive Control (GPC) is being studied to assess its potential for actively controlling the swashplate of tiltrotor aircraft to enhance aeroelastic stability in both helicopter and airplane modes of flight. This paper summarizes the exploratory numerical and experimental studies that were conducted as part of that investigation.				
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